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Duals of noncommutative supersymmetric U(1) gauge theory

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Abstract: Parent actions for component fields are utilized to derive the dual of supersymmetric U(1) gauge theory in 4 dimensions. Generalization of the Seiberg-Witten map to the component fields of noncommutative supersymmetric U(1) gauge theory is analyzed. Through this transformation we proposed parent actions for noncommutative supersymmetric U(1) gauge theory as generalization of the ordinary case. Duals of noncommutative supersymmetric U(1) gauge theory are obtained. Duality symmetry under the interchange of fields with duals accompanied by the replacement of the noncommutativity parameter $\Theta_{\mu\nu}$ with $\tilde{\Theta}_{\mu\nu} = g^2\epsilon_{\mu\nu\rho\sigma}\Theta^{\rho\sigma}$ of the non-supersymmetric case is broken at the level of actions. We proposed a noncommutative parent action for the component fields which generates actions possessing this duality symmetry.

Keywords: Duality in Gauge Field Theories, Non-Commutative Geometry.

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1. Introduction

Electric-magnetic duality invariance of Maxwell equations can be formulated at the level of actions due to a parent action which generates both the original and the dual actions. This approach was used to derive dual of noncommutative U(1) gauge theory [1] after transforming noncommutative gauge theory to commutative one by the Seiberg-Witten map [2]. Here we study dual transformations of noncommutative supersymmetric U(1) gauge theory in 4 dimensions using a similar procedure considering the component fields of superfields.

Parent action of "ordinary" supersymmetric U(1) gauge theory was formulated by superfields [3]. In terms of component fields we define two different parent actions which yield the same dual symmetric actions.

Noncommutative supersymmetric U(1) gauge theory can be defined as generalization of supersymmetric Yang-Mills gauge theory either through superfields [4] or using their component fields [5]. To derive its dual theory by performing duality transformation for the ordinary fields as in [1], one should find a transformation which generalizes the Seiberg-Witten map [2] to noncommutative supersymmetric U(1) gauge theory fields. This transformation was studied in two different ways through superfields [4, 6]. We utilize both of these approaches to define a generalization of the Seiberg-Witten map for the component fields. Then we write noncommutative supersymmetric U(1) gauge theory in terms of the component fields which are valued in commuting space-time. We only deal with the terms up to the first order in the noncommutativity parameter $\Theta_{\mu\nu}$.

Generalizing parent actions of ordinary supersymmetric gauge theory and using the map between "noncommutative" and ordinary (commutative) fields we propose two different parent actions. Both of them generate noncommutative supersymmetric U(1) gauge theory given by the component fields defined in commuting space-time. However, they yield different dual actions. At the first order in $\Theta_{\mu\nu}$ one of the dual actions does not have any contribution from the fermionic and the auxiliary fields. Moreover, it does not lead to the dual action of non-supersymmetric gauge theory of [1]. The other parent action

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generates a dual theory which embraces the results of [1]. However, this dual action is not in the same form with the noncommutative U(1) gauge theory. Thus, duality symmetry of the non-supersymmetric theory given by replacing the field strength $F^{\mu\nu}$ with the dual one $F_D^{\mu\nu}$ and $\Theta_{\mu\nu}$ with $\tilde{\Theta}_{\mu\nu} = g^2 \epsilon_{\mu\nu\rho\sigma}\Theta^{\rho\sigma}$ is not satisfied when actions are considered. We introduce a parent action for the component fields which generates actions possessing this duality symmetry. Unfortunately, it is not clear if these duality symmetric actions are supersymmetric, though they are explicitly gauge invariant.

2. Parent actions for component fields

Working in 4 dimensional Minkowski space-time and the $N = 1$ superspace $(x_\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$ we consider a general chiral superfield (not a supersymmetric field strength) $\tilde{W}_\alpha$ and a real (dual) vector field V_D to write the parent action [3]

$$I_p = \frac{1}{4g^2}\int d^4x \left(\int d^2\theta \tilde{W}^2 + \int d^2\bar{\theta}\bar{\tilde{W}}^2\right) + \frac{1}{2}\int d^4x d^4\theta \left(V_D D\tilde{W} - V_D \bar{D}\bar{\tilde{W}}\right) \tag{2.1}$$

where D_α is the supercovariant derivative. We use notations of [7]. Equation of motion with respect to the super vector field V_D leads to the supersymmetric generalization of the Bianchi identity

$$D\tilde{W} - \bar{D}\bar{\tilde{W}}|_W = 0\,. \tag{2.2}$$

Its solution is the supersymmetric field strength written in terms of the real vector superfield V as

$$W_\alpha = \frac{1}{2}\bar{D}^2 D_\alpha V\,. \tag{2.3}$$

Replacement of $\tilde{W}, \bar{\tilde{W}}$ with the solution (2.3) in the parent action (2.1), which is equivalent to perform the path integral over V_D in its partition function, leads to

$$I = \frac{1}{4g^2}\int d^4x \left(\int d^2\theta W^2 + \int d^2\bar{\theta}\bar{W}^2\right). \tag{2.4}$$

This is the action of supersymmetric U(1) gauge theory.

On the other hand, when solutions of the equations of motion with respect to $\tilde{W}_\alpha$ and $\bar{\tilde{W}}^{\dot{\alpha}}$ following from I_p are plugged into (2.1), one obtains the dual action

$$I_D = \frac{g^2}{4}\int d^4x \left(\int d^2\theta W_D^2 + \int d^2\bar{\theta}\bar{W}_D^2\right) \tag{2.5}$$

where W_D is the dual superfield strength $W_{D\alpha} = \frac{1}{2}\bar{D}^2 D_\alpha V_D$.

The original and the dual actions, (2.4) and (2.5), are in the same form except g^{-2} replaced with g^2. Thus, one can conclude that supersymmetric U(1) gauge theory possesses (S) duality symmetry.

Instead of superfields, we would like to consider duality transformations in terms of their component fields. It is straightforward to construct a general chiral superfield $\tilde{W}_\alpha$ that does not satisfy the condition (2.2) as

$$\tilde{W}_\alpha(y) = -i\lambda_\alpha(y) + \theta_\alpha \tilde{D}(y) - i\sigma_\alpha^{\mu\nu\,\beta}\theta_\beta \tilde{F}_{\mu\nu}(y) + \theta\theta\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}^{\dot{\alpha}}(y) \tag{2.6}$$

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where $y_\mu = x_\mu + i\theta\sigma_\mu\bar\theta$. Here, λ and $\bar\psi$ are two independent Weyl spinors, $\tilde F_{\mu\nu}$ is a complex antisymmetric field and $\tilde D$ is a complex scalar field. Hermitean conjugate of the chiral superfield $\tilde W_\alpha$ can be written as

$$\bar{\tilde W}^{\dot\alpha}(y^\dagger) = i\bar\lambda^{\dot\alpha}(y^\dagger) + \bar\theta^{\dot\alpha} D^\dagger(y^\dagger) + i\bar\sigma^{\mu\nu\dot\alpha}{}_{\dot\beta}\bar\theta^{\dot\beta} F^\dagger_{\mu\nu}(y^\dagger) + \bar\theta\bar\theta\bar\sigma^{\dot\alpha\alpha}_\mu \partial^\mu\psi_\alpha(y^\dagger)\,. \tag{2.7}$$

Plugging (2.6), (2.7) and the real vector superfield

$$V_D = -(\theta\sigma^\mu\bar\theta)A_{D\mu} + i\theta\theta\bar\theta\bar\lambda_D - i\bar\theta\bar\theta\theta\lambda_D + \frac{1}{2}\theta\theta\bar\theta\bar\theta D_D \tag{2.8}$$

into (2.1) the parent action in component fields is obtained

$$I_p = I_o[\tilde F, \psi, \lambda, \tilde D] + I_l \tag{2.9}$$

where we defined

$$\begin{aligned} I_o[\tilde F, \psi, \lambda, \tilde D] \equiv \frac{1}{g^2}\int d^4x \Big[&-\frac{1}{8}\tilde F^{\mu\nu}\tilde F_{\mu\nu} - \frac{i}{16}\epsilon^{\mu\nu\lambda\kappa}\tilde F_{\mu\nu}\tilde F_{\lambda\kappa} - \frac{1}{8}\tilde F^{\dagger\mu\nu}\tilde F^\dagger_{\mu\nu} + \\ &+ \frac{i}{16}\epsilon^{\mu\nu\lambda\kappa}\tilde F^\dagger_{\mu\nu}\tilde F^\dagger_{\lambda\kappa} - \frac{i}{2}\lambda\partial\!\!\!/\bar\psi - \frac{i}{2}\bar\lambda\bar{\partial\!\!\!/}\psi + \frac{1}{4}\tilde D^2 + \frac{1}{4}\tilde D^{\dagger 2}\Big] \end{aligned} \tag{2.10}$$

and the Legendre transformation term

$$\begin{aligned} I_l \equiv \frac{1}{2}\int d^4x \Big[&-i\tilde F^{\mu\nu}\partial_\mu A_{D\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa}\tilde F_{\mu\nu}\partial_\lambda A_{D\kappa} + i\tilde F^{\dagger\mu\nu}\partial_\mu A_{D\nu} + \frac{1}{2}\epsilon^{\mu\nu\lambda\kappa}\tilde F^\dagger_{\mu\nu}\partial_\lambda A_{D\kappa} + \\ &+ \frac{1}{2}\lambda_D\partial\!\!\!/\bar\psi + \frac{1}{2}\lambda\partial\!\!\!/\bar\lambda_D - \frac{1}{2}\bar\lambda_D\bar{\partial\!\!\!/}\psi - \frac{1}{2}\bar\lambda\bar{\partial\!\!\!/}\lambda_D + iD_D(\tilde D - \tilde D^\dagger)\Big]. \end{aligned} \tag{2.11}$$

We now proceed as before to derive supersymmetric U(1) gauge theory in terms of the component fields from the parent action (2.9): The equations of motion with respect to the dual vector field $A_{D\mu}$

$$\left[\frac{i}{2}(\partial_\mu\tilde F^{\mu\kappa} - \partial_\mu\tilde F^{\dagger\mu\kappa}) - \frac{1}{4}\epsilon^{\mu\nu\lambda\kappa}\partial_\lambda(\tilde F_{\mu\nu} + \tilde F^\dagger_{\mu\nu})\right]_{\tilde F = F} = 0\,. \tag{2.12}$$

lead to $F_{\mu\nu}$ which satisfy

$$F_{\mu\nu} = F^\dagger_{\mu\nu},\,, \qquad \epsilon^{\mu\nu\lambda\kappa}\partial_\lambda F_{\mu\nu} = 0\,, \tag{2.13}$$

which are solved by taking $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ which is the field strength of the vector field A_μ. When we also use the equations of motion with respect to the other dual fields

$$\partial\!\!\!/\bar\psi = \partial\!\!\!/\bar\lambda\,, \qquad \bar{\partial\!\!\!/}\psi = \bar{\partial\!\!\!/}\lambda\,, \qquad \tilde D - \tilde D^\dagger|_{\tilde D = D} = 0\,. \tag{2.14}$$

in the parent action (2.9) we obtain the supersymmetric U(1) gauge theory action in terms of component fields

$$I = \frac{1}{g^2}\int d^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{i}{2}\lambda\partial\!\!\!/\bar\lambda - \frac{i}{2}\bar\lambda\bar{\partial\!\!\!/}\lambda + \frac{1}{2}D^2\right]. \tag{2.15}$$

Similarly, we can obtain the dual action (2.5) in terms of the component fields using the equations of motion of (2.9) with respect to the fields $\tilde{F}_{\mu\nu}, \lambda, \bar{\psi}, \tilde{D}$:

$$\left(\eta^{\mu\lambda}\eta^{\nu\kappa} - \eta^{\mu\kappa}\eta^{\nu\lambda} + i\epsilon^{\mu\nu\lambda\kappa}\right) F_{\lambda\kappa} = -ig^2 \left(\eta^{\mu\lambda}\eta^{\nu\kappa} - \eta^{\mu\kappa}\eta^{\nu\lambda} + i\epsilon^{\mu\nu\lambda\kappa}\right) F_{D\lambda\kappa}, \tag{2.16}$$

$$\partial\!\!\!/\bar{\psi} = -ig^2 \partial\!\!\!/\bar{\lambda}_D\,, \qquad \bar{\partial\!\!\!/}\lambda = -ig^2 \bar{\partial\!\!\!/}\lambda_D\,, \qquad \tilde{D} = -ig^2 D_D \tag{2.17}$$

and the equations of motion with respect to $\tilde{F}^{\dagger}_{\mu\nu}, \bar{\lambda}, \psi, \tilde{D}^{\dagger}$:

$$\left(\eta^{\mu\lambda}\eta^{\nu\kappa} - \eta^{\mu\kappa}\eta^{\nu\lambda} - i\epsilon^{\mu\nu\lambda\kappa}\right) \tilde{F}^{\dagger}_{\lambda\kappa} = ig^2 \left(\eta^{\mu\lambda}\eta^{\nu\kappa} - \eta^{\mu\kappa}\eta^{\nu\lambda} - i\epsilon^{\mu\nu\lambda\kappa}\right) F_{D\lambda\kappa}\,, \tag{2.18}$$

$$\partial\!\!\!/\bar{\lambda} = ig^2 \partial\!\!\!/\bar{\lambda}_D\,, \qquad \bar{\partial\!\!\!/}\psi = ig^2 \bar{\partial\!\!\!/}\lambda_D\,, \qquad \tilde{D}^{\dagger} = ig^2 D_D\,, \tag{2.19}$$

where $F_{D\mu\nu} = \partial_\mu A_{D\nu} - \partial_\nu A_{D\mu}$. Solutions of them are plugged into (2.9) yielding the dual supersymmetric U(1) gauge theory action (2.5) in terms of the component fields

$$I_D = g^2 \int d^4x \left[-\frac{1}{4} F_D^{\mu\nu} F_{D\mu\nu} - \frac{i}{2}\lambda_D \partial\!\!\!/\bar{\lambda}_D - \frac{i}{2}\bar{\lambda}_D \bar{\partial\!\!\!/}\lambda_D + \frac{1}{2} D_D^2\right]. \tag{2.20}$$

Instead of the complex field $\tilde{F}_{\mu\nu}$ we can deal with the real antisymmetric tensor field $F_{R\mu\nu}$ from the beginning. For this case we propose

$$S_p = S_o[F_R, \psi, \lambda, \tilde{D}] + S_l\,, \tag{2.21}$$

as parent action, where

$$S_o[F_R, \psi, \lambda, \tilde{D}] \equiv \frac{1}{4g^2} \int d^4x \left[-F_R^{\mu\nu} F_{R\mu\nu} - 2i\bar{\lambda}\sigma^\mu \partial_\mu \psi - 2i\lambda\sigma^\mu \partial_\mu \bar{\psi} + \tilde{D}^2 + \tilde{D}^{\dagger 2}\right], \tag{2.22}$$

and the Legendre transformation part

$$\begin{aligned} S_l \equiv \frac{1}{2} \int d^4x \Big[&\varepsilon^{\mu\nu\rho\sigma} F_{R\mu\nu} \partial_\rho A_{D\sigma} + \lambda_D \sigma^\mu \partial_\mu \bar{\psi} + \bar{\lambda}_D \bar{\sigma}^\mu \partial_\mu \lambda - \\ &- \lambda_D \sigma^\mu \partial_\mu \bar{\lambda} - \bar{\lambda}_D \bar{\sigma}^\mu \partial_\mu \psi + iD_D \left(\tilde{D} - \tilde{D}^{\dagger}\right)\Big]. \end{aligned} \tag{2.23}$$

The equations of motions with respect to the dual fields $A_D,\ \lambda_D,\ \bar{\lambda}_D,\ D_D$,

$$\varepsilon^{\mu\nu\rho\sigma} \partial_\nu F_{R\rho\sigma}|_{F_R = F} = 0\,, \tag{2.24}$$

$$\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\psi}^{\dot{\alpha}} - \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \bar{\lambda}^{\dot{\alpha}} = 0\,, \tag{2.25}$$

$$\bar{\sigma}^{\dot{\alpha}\alpha}_\mu \partial^\mu \lambda_\alpha - \bar{\sigma}^{\dot{\alpha}\alpha}_\mu \partial^\mu \psi_\alpha = 0\,, \tag{2.26}$$

$$\left(\tilde{D} - \tilde{D}^{\dagger}\right)_{\tilde{D} = D} = 0\,, \tag{2.27}$$

are solved in terms of the field strength $F_{\mu\nu}$ and real scalar field D. These solutions when used in the parent action (2.21) yield the supersymmetric U(1) gauge theory (2.15).

The equations of motions with respect to the fields $F_{R\mu\nu}, \lambda, \psi, \bar{\lambda}, \tilde{D}, \bar{\psi}, \tilde{D}^{\dagger}$ are

$$\begin{aligned} &-\frac{1}{g^2} F_R^{\mu\nu} + \varepsilon^{\mu\nu\rho\sigma} \partial_\rho A_{D\sigma} = 0 \\ &\frac{1}{g^2} \tilde{D}^{\dagger} - iD_D = 0\,, \qquad \frac{1}{g^2} \tilde{D} + iD_D = 0\,, \\ &\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \left(-\frac{i}{g^2} \bar{\psi}^{\dot{\alpha}} + \bar{\lambda}^{\dot{\alpha}}_D\right) = 0\,, \qquad \bar{\sigma}^{\mu\dot{\alpha}\alpha} \partial_\mu \left(-\frac{i}{g^2} \psi_\alpha - \lambda_{D\alpha}\right) = 0\,, \\ &\partial_\mu \left(-\frac{i}{g^2} \bar{\lambda}_{\dot{\alpha}} + \bar{\lambda}_{D\dot{\alpha}}\right) \bar{\sigma}^{\mu\dot{\alpha}\alpha} = 0\,, \qquad \partial_\mu \left(-\frac{i}{g^2} \lambda^\alpha + \lambda^\alpha_D\right) \sigma^\mu_{\alpha\dot{\alpha}} = 0\,. \end{aligned} \tag{2.28}$$

Solving these equations for the dual fields and substituting them in the parent action (2.21) yield the dual of action of $N = 1$ supersymmetric U(1) gauge theory (2.20).

We conclude that both of the parent actions (2.9) and (2.21) generate supersymmetric U(1) gauge theory and its dual.

3. Supersymmetric Seiberg-Witten map

Noncommutativity is introduced through the star product

$$* \equiv \exp \frac{i\theta^{\mu\nu}}{2}\left(\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu - \overleftarrow{\partial}_\nu \overrightarrow{\partial}_\mu\right), \tag{3.1}$$

where x_μ are space-time coordinates and $\theta^{\mu\nu}$ is an antisymmetric and constant real parameter. Now, the coordinates x^μ satisfy the Moyal bracket

$$x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu}\,. \tag{3.2}$$

We assume that surface terms are vanishing, so that the following properties are satisfied

$$\begin{aligned}\int d^4x f(x) * g(x) &= \int d^4x f(x) g(x),\\ \int d^4x f(x) * g(x) * h(x) &= \int d^4x (f(x) * g(x)) h(x) = \int d^4x f(x)(g(x) * h(x))\,.\end{aligned}$$

Generalization of the Seiberg-Witten map to noncommutative supersymmetric gauge theories can be formulated in some different ways. One of these is to generalize the definition of the map between the noncommutative gauge field $\widehat{A}$, noncommutative gauge parameter $\widehat{\lambda}$ and the ordinary ones A, λ to $\widehat{V}(V), \widehat{\Lambda}(\Lambda, V)$. Here V is a vector superfield, Λ is a chiral superfield and $\widehat{V}$ and $\widehat{\Lambda}$ are corresponding "noncommutative superfields"[6]. Infinitesimal gauge transformation of the noncommutative supervector field $\widehat{V}$ is defined by

$$\widehat{\delta}_{\widehat{\Lambda}} \widehat{V} = i(\widehat{\Lambda} - \widehat{\bar{\Lambda}}) - \frac{i}{2}\left[\left(\widehat{\Lambda} + \widehat{\bar{\Lambda}}\right) * \widehat{V} - \widehat{V} * \left(\widehat{\Lambda} + \widehat{\bar{\Lambda}}\right)\right]. \tag{3.3}$$

It has the properties of a non-abelian gauge transformation, although the ordinary vector field V gauge transforms as

$$\delta_\Lambda V = i(\Lambda - \bar{\Lambda})\,. \tag{3.4}$$

Supersymmetric Seiberg-Witten map is defined as

$$\widehat{V}(V) + \widehat{\delta}_{\widehat{\Lambda}} \widehat{V}(V) = \widehat{V}(V + \delta_\Lambda V)\,. \tag{3.5}$$

In [6] a solution of this equation is given in terms of superfields. However, it is nonlocal and do not yield the original solution of Seiberg and Witten [2]

$$\widehat{A}_\mu = A_\mu - \frac{1}{2}\Theta^{kl}(A_k \partial_l A_\mu + A_k F_{l\mu})\,, \tag{3.6}$$

at the first order in the noncommutativity parameter $\Theta^{\mu\nu}$.

On the other hand the approach suggested in [4] is to generalize the solution of Seiberg and Witten (3.6) to supersymmetric case as

$$\begin{aligned}\widehat{V}(V) &= \frac{1}{2}V + a\Theta^{\mu\nu}\partial_\mu\sigma_\nu^{\alpha\dot{\alpha}}[D_\alpha, D_{\dot{\alpha}}]V + b\sigma_{\mu\nu}^{\alpha\beta}\Theta^{\mu\nu}D_\alpha V\bar{D}^2 D_\beta V + \\ &\quad + c\sigma_{\mu\nu}^{\alpha\beta}\Theta^{\mu\nu}V\bar{D}^2 D_\alpha D_\beta V + \text{c.c.}\,, \qquad (3.7)\\ \widehat{\Lambda}(\Lambda, V) &= \Lambda + d\bar{D}^2\left(\sigma_{\mu\nu}^{\alpha\beta}\Theta_{\mu\nu}D_\alpha D_\beta V\right), \qquad (3.8)\end{aligned}$$

where a, b, c, d are some constants which should be fixed using (3.6). Though, it can be solved directly it is cumbersome. Indeed, its solution is not presented in [4].

We would like to obtain a generalization of the Seiberg-Witten map to supersymmetric U(1) gauge theory in terms of the components of the superfield V. This will be performed utilizing both of the methods mentioned above. We adopt the definition (3.5) for supersymmetric Seiberg-Witten map but solve it for components of the superfield V keeping the original solution (3.6).

The vector superfield V in Wess-Zumino gauge and chiral and anti-chiral superfields Λ and $\bar{\Lambda}$, respectively, are given as

$$\begin{aligned}V &= -(\theta\sigma^\mu\bar{\theta})A_\mu + i\theta\theta\bar{\theta}\bar{\lambda} - i\bar{\theta}\bar{\theta}\theta\lambda + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D\,, \qquad (3.9)\\ \Lambda &= \beta + i(\theta\sigma^\mu\bar{\theta})\partial_\mu\beta + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2\beta + \sqrt{2}\theta\kappa - \frac{i}{\sqrt{2}}\theta\theta\partial_\mu\kappa\sigma^\mu\bar{\theta} + \theta\theta f\,, \qquad (3.10)\\ \bar{\Lambda} &= \beta^* - i(\theta\sigma^\mu\bar{\theta})\partial_\mu\beta^* + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^2\beta^* + \sqrt{2}\bar{\theta}\bar{\kappa} + \frac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^\mu\partial_\mu\bar{\kappa} + \bar{\theta}\bar{\theta}f^*\,. \qquad (3.11)\end{aligned}$$

Noncommuting superfields $\widehat{V}, \widehat{\Lambda}, \widehat{\bar{\Lambda}}$ can be written in the same form in terms of their components. At the first order in $\Theta^{\mu\nu}$ let us denote the noncommutative fields as $\widehat{V} = V + V_{(1)}$, $\widehat{\Lambda} = \Lambda + \Lambda_{(1)}$, $\widehat{\bar{\Lambda}} = \bar{\Lambda} + \bar{\Lambda}_{(1)}$ and plug them into the definition (3.5). This will yield some equations for component fields by matching the same θ order terms. In fact, the equations including only components of the superfields Λ and $\bar{\Lambda}$ are

$$\begin{aligned}\beta_{(1)} - \beta^*_{(1)} &= 0\,, \qquad (3.12)\\ f_{(1)} = f^*_{(1)} = \kappa_{(1)} = \bar{\kappa}_{(1)} &= 0\,. \qquad (3.13)\end{aligned}$$

Moreover, there are the equations

$$\begin{aligned}A_{(1)\mu}(V_i + \delta V_i) - A_{(1)\mu}(V_i) - \partial_\mu\beta &= -\Theta^{\nu\rho}\partial_\nu A_\mu\partial_\rho\beta\,, \qquad (3.14)\\ \lambda_{(1)}(V_i + \delta V_i) - \lambda_{(1)}(V_i) &= -\Theta^{\nu\rho}\partial_\nu\lambda\partial_\rho\beta\,, \qquad (3.15)\\ \bar{\lambda}_{(1)}(V_i + \delta V_i) - \bar{\lambda}_{(1)}(V_i) &= -\Theta^{\nu\rho}\partial_\nu\bar{\lambda}\partial_\rho\beta\,, \qquad (3.16)\\ D_{(1)}(V_i + \delta V_i) - D_{(1)}(V_i) &= -\Theta^{\nu\rho}\partial_\nu D\partial_\rho\beta\,, \qquad (3.17)\end{aligned}$$

where V_i denotes the component fields.

Obviously, one can write (3.5) in terms of a general vector superfield instead of choosing the Wess-Zumino gauge (3.10), which would have drastically changed the equations for component fields. However, we prefer to choose V as (3.10), so that, we deal with the equations (3.12)–(3.17) as defining supersymmetric Seiberg-Witten map.

One can solve these equations and get the noncommutative fields in terms of the ordinary ones at the first order in $\Theta^{\mu\nu}$ as

$$\widehat{A}_\mu = A_\mu - \frac{1}{2}\Theta^{\nu\rho}(A_\nu\partial_\rho A_\mu + A_\nu F_{\rho\mu})\,, \tag{3.18}$$

$$\widehat{\lambda} = \lambda - \Theta^{\nu\rho}\partial_\nu\lambda A_\rho\,, \tag{3.19}$$

$$\widehat{\bar{\lambda}} = \bar{\lambda} - \Theta^{\nu\rho}\partial_\nu\bar{\lambda} A_\rho\,, \tag{3.20}$$

$$\widehat{D} = D - \Theta^{\nu\rho}\partial_\nu D A_\rho\,. \tag{3.21}$$

Eqs. (3.18) and (3.19) are also found in [8] considering deformations of supersymmetric Yang-Mills theory while preserving supersymmetry.

We also should define $\widehat{\psi}$ which is the noncommutative component field resembling ψ needed to define a parent action to obtain duality transformation. We define supersymmetric Seiberg-Witten map of $\widehat{\psi}$ as

$$\widehat{\psi} = \psi - \Theta^{\nu\rho}\partial_\nu\psi A_\rho \tag{3.22}$$

$$\widehat{\bar{\psi}} = \bar{\psi} - \Theta^{\nu\rho}\partial_\nu\bar{\psi} A_\rho, \tag{3.23}$$

which are consistent with (2.14).

4. Duals of noncommutative supersymmetric U(1) gauge theory

Noncommutative generalization of supersymmetric U(1) gauge theory [5] can be written in terms of the so called noncommuting component fields, although they satisfy the usual (anti)commutation relations, by the star product (3.1) as

$$S_{NC} = \frac{1}{2g^2}\int d^4x\left[-\frac{1}{2}\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu} - i\widehat{\bar{\lambda}}\bar{\sigma}^\mu\widehat{D}_\mu * \widehat{\lambda} - i\widehat{\lambda}\sigma^\mu\widehat{D}_\mu * \widehat{\bar{\lambda}} + \widehat{D}\widehat{D}\right], \tag{4.1}$$

where $\widehat{D}_\mu * \widehat{\lambda} = \partial_\mu\widehat{\lambda} + i(\widehat{A}_\mu * \widehat{\lambda} - \widehat{\lambda} * \widehat{A}_\mu)$ and the noncommutative field strength is $\widehat{F}_{\mu\nu} = \partial_\mu\widehat{A}_\nu - \partial_\nu\widehat{A}_\mu + i(\widehat{A}_\mu * \widehat{A}_\nu - \widehat{A}_\nu * \widehat{A}_\mu)$. It is invariant under the supersymmetry transformations given by the fermionic constant spinor parameter ξ as

$$\delta_\xi\widehat{A}_\mu = i\xi\sigma^\mu\widehat{\bar{\lambda}} + i\bar{\xi}\bar{\sigma}^\mu\widehat{\lambda}\,, \tag{4.2}$$

$$\delta_\xi\widehat{\lambda} = \sigma^{\mu\nu}\xi\widehat{F}_{\mu\nu} + i\xi\widehat{D}\,, \tag{4.3}$$

$$\delta_\xi\widehat{D} = \bar{\xi}\bar{\sigma}^\mu\widehat{D}_\mu * \widehat{\lambda} - \xi\sigma^\mu\widehat{D}_\mu * \widehat{\bar{\lambda}}\,. \tag{4.4}$$

Making use of the generalization of Seiberg-Witten map to the supersymmetric case (3.18)–(3.21) we write, up to the first order in Θ, the action of noncommutative supersymmetric U(1) gauge theory (4.1) in terms of the ordinary component fields as

$$S_{NC}[F,\lambda,D,\Theta] = \int d^4x\Big\{-\frac{1}{4g^2}\left(F^{\mu\nu}F_{\mu\nu} + 2\Theta^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} - \frac{1}{2}\Theta^{\mu\nu}F_{\nu\mu}F_{\rho\sigma}F^{\sigma\rho}\right) -$$
$$- \frac{i}{g^2}\left[\frac{1}{2}\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda + \Theta^{\mu\nu}\left(\frac{1}{4}\bar{\lambda}\bar{\sigma}^\rho\partial_\rho\lambda F_{\mu\nu} + \frac{1}{2}\bar{\lambda}\bar{\sigma}^\rho\partial_\mu\lambda F_{\nu\rho}\right) +\right.$$

$$+\frac{1}{2}\lambda\sigma^\mu\partial_\mu\bar{\lambda}+\Theta^{\mu\nu}\left(\frac{1}{4}\lambda\sigma^\rho\partial_\rho\bar{\lambda}F_{\mu\nu}+\frac{1}{2}\lambda\sigma^\rho\partial_\mu\bar{\lambda}F_{\nu\rho}\right)\Bigg]+$$
$$+\frac{1}{2g^2}\left(D^2+\frac{1}{2}\Theta^{\mu\nu}D^2F_{\mu\nu}\right)\Bigg\}. \quad (4.5)$$

Obviously, when we write this action we set the surface terms to zero while performing partial integrals. The same action was also obtained in [9] using a completely different approach.

Supersymmetry transformations which leave (4.5) invariant can be read from (4.2)–(4.4) as

$$\begin{aligned}\delta_\xi A_\mu &= i\xi\sigma_\mu\bar{\lambda}+i\bar{\xi}\bar{\sigma}_\mu\lambda-i\Theta^{\rho\kappa}(\xi\sigma_\rho\bar{\lambda}+\bar{\xi}\bar{\sigma}_\rho\lambda)\left(\frac{1}{2}F_{\kappa\mu}+\frac{1}{2}\partial_\kappa A_\mu\right)-\\ &\quad -i\Theta^{\rho\kappa}\frac{1}{2}(\xi\sigma_\rho\partial_\mu\bar{\lambda}+\bar{\xi}\bar{\sigma}_\rho\partial_\mu\lambda)A_\kappa\,, \qquad (4.6)\\ \delta_\xi\lambda &= \sigma^{\mu\nu}\xi F_{\mu\nu}+i\xi D+\Theta^{\rho\kappa}\partial_\rho\lambda(i\xi\sigma_\kappa\bar{\lambda}+i\bar{\xi}\bar{\sigma}_\kappa\lambda)+\\ &\quad +\Theta^{\rho\kappa}i\sigma^{\mu\nu}\xi F_{\mu\rho}F_{\nu\kappa}\,, \qquad (4.7)\\ \delta_\xi D &= \bar{\xi}\bar{\sigma}^\mu\partial_\mu\lambda-\xi\sigma^\mu\partial_\mu\bar{\lambda}-i\Theta^{\rho\kappa}(\xi\sigma_\rho\bar{\lambda}+\bar{\xi}\bar{\sigma}_\rho\lambda)\partial_\kappa D+\\ &\quad +\Theta^{\rho\kappa}\xi\sigma^\mu F_{\rho\mu}\partial_\kappa\bar{\lambda}-\Theta^{\rho\kappa}\bar{\xi}\bar{\sigma}^\mu F_{\rho\mu}\partial_\kappa\lambda\,. \qquad (4.8)\end{aligned}$$

We would like to generalize the parent actions of the ordinary supersymmetric gauge theory (2.9) and (2.21) to the noncommutative case. To this aim let us first take $\widehat{F}_{\mu\nu}$ complex and deal with

$$\begin{aligned}I_{oNC} = -\frac{1}{2g^2}\int d^4x\Bigg[&\frac{1}{4}\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu}+\frac{i}{8}\epsilon^{\mu\nu\rho\sigma}\widehat{F}_{\mu\nu}\widehat{F}_{\rho\sigma}+\frac{1}{4}\widehat{F}^{\dagger\mu\nu}\widehat{F}^\dagger_{\mu\nu}+\frac{i}{8}\epsilon^{\mu\nu\rho\sigma}\widehat{F}^\dagger_{\mu\nu}\widehat{F}^\dagger_{\rho\sigma}+\\ &+i\widehat{\lambda}\sigma^\mu\widehat{D}_\mu*\widehat{\bar{\psi}}+i\widehat{\bar{\lambda}}\bar{\sigma}^\mu\widehat{D}_\mu*\widehat{\psi}-\frac{1}{2}\widehat{D}^2-\frac{1}{2}\widehat{D}^{\dagger2}\Bigg]. \qquad (4.9)\end{aligned}$$

It is possible to discuss supersymmetry and gauge transformations of (4.9), however, it is not needed for the purposes of this work.

Although the transformations (3.18)–(3.23) are derived for a real vector superfield, we suppose that they are also valid for complex fields. We perform the transformations (3.18)–(3.23) and their complex conjugates to write (4.9) as

$$\begin{aligned}I_{oNC}[F,\lambda,\psi,D] &= I_o[F,\lambda,\psi,D]-\\ &-\frac{\Theta^{\mu\nu}}{g^2}\int d^4x\Bigg[\frac{1}{4}F^{\rho\sigma}F_{\rho\mu}F_{\nu\sigma}+\frac{1}{16}F_{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}+\\ &\qquad+\frac{i}{8}\epsilon^{\lambda\kappa\rho\sigma}F_{\lambda\kappa}F_{\rho\mu}F_{\nu\sigma}+\frac{i}{32}\epsilon^{\lambda\kappa\rho\sigma}F_{\mu\nu}F_{\lambda\kappa}F_{\rho\sigma}+\\ &\qquad+\frac{i}{4}\lambda\sigma^\rho\partial_\rho\bar{\psi}F_{\mu\nu}-\frac{i}{2}\lambda\sigma^\rho\partial_\nu\bar{\psi}F_{\mu\rho}-\frac{1}{4}F_{\mu\nu}D^2+\text{c.c.}\Bigg], \qquad (4.10)\end{aligned}$$

where I_o is defined in (2.10). We define the parent action

$$I_P = I_{oNC}[\tilde{F},\lambda,\psi,\tilde{D}]+I_l\,, \qquad (4.11)$$

where I_l is given in (2.11). We would like to emphasize that $\tilde{F}_{\mu\nu}$ is not a field strength but a complex, antisymmetric field. When the solutions of the equations of motion with respect to dual fields (2.12)–(2.14) are used in the parent action, it leads to the noncommutative supersymmetric U(1) gauge theory action (4.5). However, when the equations of motion with respect to the fields $\tilde{F}$, λ, ψ, $\tilde{D}$ and their complex conjugates are solved and used in the parent action (4.10) one finds

$$I_{DNC} = I_D + \frac{g^4}{4}\Theta^{\mu\nu}\int d^4x\epsilon^{\lambda\kappa\rho\sigma}\left[F_{D\lambda\kappa}F_{D\rho\mu}F_{D\nu\sigma} + \frac{1}{4}F_{D\mu\nu}F_{D\lambda\kappa}F_{D\rho\sigma}\right], \tag{4.12}$$

where F_D is the field strength of A_D. Obviously, we cannot define any duality symmetry between (4.5) and (4.12). The latter does not possess any contribution in terms of the fields λ, D at the first order in $\Theta_{\mu\nu}$.

As the other possibility, let us take $\widehat{F}_{\mu\nu}$ real and deal with

$$S_{oNC} = \int d^4x\left[-\frac{1}{4g^2}\widehat{F}^{\mu\nu}\widehat{F}_{\mu\nu} - \frac{i}{2g^2}\widehat{\bar{\lambda}}\bar{\sigma}^\mu\widehat{D}_\mu * \widehat{\psi} - \frac{i}{2g^2}\widehat{\lambda}\sigma^\mu\widehat{D}_\mu * \widehat{\bar{\psi}} + \frac{1}{2g^2}\widehat{D}\widehat{D}^\dagger\right]. \tag{4.13}$$

Through the supersymmetric Seiberg-Witten map (3.18)–(3.23), we write the action (4.13) as

$$\begin{aligned}S_{oNC}[F,\lambda,\psi,D] = \int d^4x\Big\{&-\frac{1}{4g^2}\left(F^{\mu\nu}F_{\mu\nu} + 2\Theta^{\mu\nu}F_{\nu\rho}F^{\rho\sigma}F_{\sigma\mu} - \frac{1}{2}\Theta^{\mu\nu}F_{\nu\mu}F_{\rho\sigma}F^{\sigma\rho}\right) - \\ &-\frac{i}{2g^2}\left(\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\psi + \Theta^{\mu\nu}\bar{\lambda}\bar{\sigma}^\rho\partial_\mu\psi F_{\nu\rho} + \frac{1}{2}\Theta^{\mu\nu}\bar{\lambda}\bar{\sigma}^\rho\partial_\rho\psi F_{\mu\nu}\right) - \\ &-\frac{i}{2g^2}\left(\lambda\sigma^\mu\partial_\mu\bar{\psi} + \Theta^{\mu\nu}\lambda\sigma^\rho\partial_\mu\bar{\psi}F_{\nu\rho} + \frac{1}{2}\Theta^{\mu\nu}\lambda\sigma^\rho\partial_\rho\bar{\psi}F_{\mu\nu}\right) + \\ &+\frac{1}{4g^2}\left[D^2 + D^{\dagger 2} + \frac{1}{2}\Theta^{\mu\nu}\left(D^2 + D^{\dagger 2}\right)F_{\mu\nu}\right]\Big\}.\end{aligned} \tag{4.14}$$

Now, we define the parent action as

$$S_P = S_{oNC}[F_R,\lambda,\psi,\tilde{D}] + S_l \tag{4.15}$$

where as before $F_{R\mu\nu}$ denotes an antisymmetric real field and the Legendre transformation part S_l is given in (2.23).

Equations of motion with respect to the dual fields $A_D, \lambda_D, \bar{\lambda}, D_D$ are given as before by (2.24)–(2.27). Plugging their solutions into S_{oNC} leads to the noncommutative supersymmetric U(1) gauge theory (4.5).

Equations of motion with respect to the other fields are

$$\begin{aligned}&-\frac{1}{g^2}F_R^{\mu\nu} - \frac{1}{g^2}\Theta^{\rho[\mu}F_R^{\nu]\sigma}F_{R\sigma\rho} - \frac{1}{2g^2}\Theta^{\rho\sigma}F_{R\sigma[\mu}F_{R\nu]\rho} + \frac{1}{4g^2}\Theta^{\mu\nu}F_{R\rho\sigma}F_R^{\rho\sigma} + \\ &+\frac{1}{2g^2}\Theta^{\rho\sigma}F_{R\rho\sigma}F_R^{\mu\nu} - \frac{i}{2g^2}(\Theta^{\rho\mu}\bar{\lambda}\bar{\sigma}^\nu - \Theta^{\rho\nu}\bar{\lambda}\bar{\sigma}^\mu)\partial_\rho\psi - \frac{i}{2g^2}\Theta^{\mu\nu}(\bar{\lambda}\bar{\sigma}^\rho\partial_\rho\psi) - \\ &\qquad -\frac{i}{2g^2}(\Theta^{\rho\mu}\lambda\sigma^\nu - \Theta^{\rho\nu}\lambda\sigma^\mu)\partial_\rho\bar{\psi} - \frac{i}{2g^2}\Theta^{\mu\nu}\lambda\sigma^\rho\partial_\rho\bar{\psi} +\end{aligned}$$

$$+\frac{1}{4g^2}\Theta^{\mu\nu}(\tilde{D}^2+\tilde{D}^{\dagger 2})-\varepsilon^{\mu\nu\rho\sigma}\partial_\rho A_{D\sigma}=0\,, \qquad (4.16)$$

$$-\frac{i}{2g^2}\sigma^\mu\partial_\mu\bar{\psi}-\frac{i}{4g^2}\Theta^{\mu\nu}\sigma^\rho\partial_\rho\bar{\psi}F_{R\mu\nu}-\frac{i}{2g^2}\Theta^{\mu\nu}\sigma^\rho\partial_\mu\bar{\psi}F_{R\nu\rho}+\frac{1}{2}\sigma^\mu\partial_\mu\bar{\lambda}_D=0\,, \qquad (4.17)$$

$$-\frac{i}{2g^2}\bar{\sigma}^\mu\partial_\mu\psi-\frac{i}{4g^2}\Theta^{\mu\nu}\bar{\sigma}^\rho\partial_\rho\psi F_{R\mu\nu}-\frac{i}{2g^2}\Theta^{\mu\nu}\bar{\sigma}^\rho\partial_\mu\psi F_{R\nu\rho}-\frac{1}{2}\bar{\sigma}^\mu\partial_\mu\lambda_D=0\,, \qquad (4.18)$$

$$-\partial_\mu\left[\frac{i}{2g^2}\bar{\lambda}\bar{\sigma}^\mu-\frac{i}{4g^2}\Theta^{\rho\nu}\bar{\lambda}\bar{\sigma}^\mu F_{R\rho\nu}-\frac{i}{2g^2}\Theta^{\mu\nu}\bar{\lambda}\bar{\sigma}^\rho F_{R\nu\rho}-\frac{1}{2}\bar{\lambda}_D\bar{\sigma}_\mu\right]=0\,, \qquad (4.19)$$

$$\partial_\mu\left[-\frac{i}{2g^2}\lambda\sigma^\mu-\frac{i}{4g^2}\Theta^{\rho\nu}\lambda\sigma^\mu F_{R\rho\nu}-\frac{i}{2g^2}\Theta^{\mu\nu}\lambda\sigma^\rho F_{R\nu\rho}+\frac{1}{2}\lambda_D\sigma^\mu\right]=0\,, \qquad (4.20)$$

$$\frac{1}{2g^2}\tilde{D}+\frac{1}{4g^2}\Theta^{\mu\nu}\tilde{D}F_{R\mu\nu}+\frac{i}{4}D_D=0\,, \qquad (4.21)$$

$$\frac{1}{2g^2}\tilde{D}^\dagger+\frac{1}{4g^2}\Theta^{\mu\nu}\tilde{D}^\dagger F_{R\mu\nu}-\frac{i}{4}D_D=0\,. \qquad (4.22)$$

We solve these equations for $F_R,\psi,\lambda,\tilde{D}$ and plug the solutions into (4.15) to obtain the dual action

$$\begin{aligned}S_{NCD}=\int d^4x\Big[&-\frac{g^2}{4}\left(F_D^{\mu\nu}F_{D\mu\nu}+2\tilde{\Theta}^{\mu\nu}F_{D\nu\rho}F_D^{\rho\sigma}F_{D\sigma\mu}-\frac{1}{2}\tilde{\Theta}^{\mu\nu}F_{D\nu\mu}F_{D\rho\sigma}F^{D\sigma\rho}\right)-\\&-ig^2\left(\frac{1}{2}\lambda_D\sigma^\mu\partial_\mu\bar{\lambda}_D+\frac{1}{2}\bar{\lambda}_D\bar{\sigma}^\mu\partial_\mu\lambda_D+\frac{1}{4}\tilde{\Theta}^{\mu\nu}\lambda_D\sigma_\mu\partial^\rho\bar{\lambda}_D F_{D\rho\nu}+\right.\\&\left.+\frac{1}{4}\tilde{\Theta}^{\mu\nu}\bar{\lambda}_D\bar{\sigma}_\mu\partial^\rho\lambda_D F_{D\rho\nu}\right)+\frac{g^2}{2}\left(D_D^2+\frac{1}{2}\tilde{\Theta}^{\mu\nu}D_D^2F_{D\mu\nu}\right)\Big], \qquad (4.23)\end{aligned}$$

where

$$\tilde{\Theta}^{\mu\nu}\equiv g^2\epsilon^{\mu\nu\rho\sigma}\Theta_{\rho\sigma}\,, \qquad (4.24)$$

When the fermionic and auxiliary fields λ_D, D_D set equal to zero one obtains the result of [1]: There is a duality symmetry under the replacement of A^μ with A^μ_D and $\Theta_{\mu\nu}$ with $\tilde{\Theta}_{\mu\nu}$. Unfortunately, this symmetry accompanied by the replacement of λ, D with λ_D, D_D, cease to exist between the noncommutative supersymmetric action (4.5) and its dual (4.23). Inspecting the terms which obstruct the duality symmetry we can find actions in terms of the component fields which possess this symmetry. Let us define the action

$$\Sigma(\Theta,F,\lambda,\bar{\lambda},D)=S_{NC}-\frac{i}{g^2}\int d^4x\Theta^{\mu\nu}\left(\lambda\sigma_\mu\partial^\rho\bar{\lambda}+\bar{\lambda}\bar{\sigma}_\mu\partial^\rho\lambda\right)F_{\rho\nu}\,, \qquad (4.25)$$

which can be obtained from the parent action

$$\Sigma_P=S_P-\frac{i}{2g^2}\int d^4x\Theta^{\mu\nu}\left(\psi\sigma_\mu\partial^\rho\bar{\lambda}+\bar{\psi}\bar{\sigma}_\mu\partial^\rho\lambda+\lambda\sigma_\mu\partial^\rho\bar{\psi}+\bar{\lambda}\bar{\sigma}_\mu\partial^\rho\psi\right)F_{R\rho\nu}\,, \qquad (4.26)$$

when the solutions of equations of motion with respect to dual fields A_D,λ_D,D_D are plugged into it. Now, the dual theory which follows from (4.26) can be shown to be

$$\Sigma_D=g^4\Sigma(\tilde{\Theta},F_D,\lambda_D,\bar{\lambda}_D,D_D)\,. \qquad (4.27)$$

Therefore we conclude that the action (4.25) possesses the duality symmetry when the original fields are substituted by the dual ones and the noncommutativity parameter Θ is replaced with $\tilde{\Theta}$. However, whether the action (4.25) is supersymmetric or not is an open question. However, it is explicitly gauge invariant.

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